Correction to “A Real Algebraic Vector Bundle is Strongly Algebraic whenever its Total Space is Affine”

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M. Coste pointed out to me that the proof of the Lemma of [2] is incomplete. Indeed, with notation as in [2], the natural map from an \(\mathcal{R}(X)\)-module \(M\) into the \(\mathcal{R}(X)\)-module of global sections of the sheaf \(M\) is not necessarily surjective. To convince oneself, one takes \(X = \mathbb{R}^2\) and \(M = \mathcal{R}(X)/(x^2(x-1)^2 + y^2)\). Let \(U = X \setminus \{(0,0)\}\) and \(V = X \setminus \{(1,0)\}\). Consider the exact sequence associated to the covering \(\{U, V\}\) of \(X\):

\[0 \to \Gamma(X, M) \to \Gamma(U, \overline{M}) \times \Gamma(V, \overline{M}) \to \Gamma(U \cap V, \overline{M})\]

Since \(x^2(x-1)^2 + y^2\) is invertible on \(U \cap V\), the group \(\Gamma(U \cap V, \overline{M})\) is 0. Hence, the map from \(\Gamma(X, \overline{M})\) into \(\Gamma(U, \overline{M}) \times \Gamma(V, \overline{M})\) is an isomorphism. Since the natural map from \(M \times M\) into the product \(\Gamma(U, \overline{M}) \times \Gamma(V, \overline{M})\) is injective, the natural map from \(M\) into \(\Gamma(X, \overline{M})\) is not surjective.

The preceding observation shows that—to say the least—an argument is missing in the proof of the Lemma of [2]. Indeed, in that proof, I claimed implicitly that the global sections \(s_U\) of \(M\) are elements of \(M\).

Now, the Lemma is only invoked at the end of the proof of the Theorem of [2]; at the point where we know that \(\Gamma(\cdot, \xi^\gamma)\) is isomorphic to \((I/I^2) \otimes_{\mathcal{R}(X)} \mathcal{R}_X\). This fact already implies that \(\xi\) is strongly algebraic. Indeed, for each maximal ideal \(m\) of \(\mathcal{R}(X)\) the \(\mathcal{R}(X)_m\)-module \((I/I^2)_m\) is free of rank \(n\), where \(n\) is the rank of \(\xi\). Moreover, \(I/I^2\) is of finite type as \(\mathcal{R}(X)\)-module, \(\mathcal{R}(E)\) being Noetherian. Hence ([1], Theorem II.5.2.2) the \(\mathcal{R}(X)\)-module \(I/I^2\) is projective and of finite type. This implies that \(\xi^\gamma\) is strongly algebraic. It follows that \(\xi\) is strongly algebraic. \(\square\)

1991 Mathematics Subject Classification. 14P05.
References


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